

## Structural Configurations, Completeness Dynamics, and Geometric Invariants of Fuzzy Hilbert Spaces

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### Abstract

This research monograph provides a comprehensive mathematical consolidation of the structural configurations, topological completeness, geometric formulations, and foundational dualities characterizing Fuzzy Hilbert Spaces ( $\mathcal{H}_F$ ). By extending classical inner product structures into uncertain functional topologies governed by continuous triangular norms ( $t$ -norms), we systematically evaluate the analytical properties of Fuzzy Normed Spaces (FNS) and Fuzzy Inner Product Spaces (FIPS). Key geometric dualities—including the fuzzy analogues of the Pythagorean theorem, the parallelogram law, the Best Approximation theorem, and the Riesz Representation Theorem—are thoroughly developed. The investigation demonstrates how analytical completeness ensures structural invariance, establishing a rigorous topological architecture for soft computing, variational dualities, and quantum modeling under epistemic uncertainty.

### Keywords

Best Approximation Theory, Continuous  $t$ -Norms, Fuzzy Hilbert Spaces, Fuzzy Orthogonality.