A Unified Nonlocal Strain Gradient Elasticity Model for Metamaterials Nano-Beams

Aurora Angela Pisano
University Mediterranea of Reggio Calabria, Italy

Paolo Fuschi
University Mediterranea of Reggio Calabria, Italy

Abstract

Size-dependent effects observed in small-scale structures, like metamaterial nano-beams, require advanced models to accurately predict their mechanical behavior. Classical nonlocal elasticity beam theories based on strain integrals typically lead to stiffness softening, whereas strain gradient elasticity theories predict stiffness hardening. To capture both phenomena within a unified framework, this work introduces a nonlocal constitutive strain gradient elasticity beam model that combines two local/nonlocal phases: one driven by strain and the other by strain gradient, incorporating internal length scale parameters and attenuation kernels. The model admits equivalent integral and differential formulations, ensuring consistency and eliminating paradoxes often reported in nonlocal beam theories. The governing integro-differential equation is of sixth order with six boundary conditions (four classical and two gradient), while its differential counterpart is of eighth order, requiring two additional nonlocality conditions. A two-step solution procedure is implemented to reduce computational complexity, transforming the original problem into the solution of fourth-order ordinary differential equations. Numerical analyses on benchmark nano-beams demonstrate the model's ability to predict both hardening and softening behaviors, depending on the dominant length scale parameters. The results confirm that the proposed approach provides a well-posed boundary value problem and offers a versatile tool for modeling size effects in nanostructures and metamaterials. This makes the theory particularly promising for applications where accurate prediction of size-dependent mechanical response is essential.

Keywords

Nonlocal elasticity, size effects, metamaterial nano-beams, strain gradient elasticity.