Analytic Approximation of Resonance Frequencies of Soft Cells

Bagyzhan Bagymkyzy
Ph.D. Student, Department of Mathematics, Nazarbayev University, Astana, Kazakhstan

Dongming Wei
Professor, Department of Mathematics, Nazarbayev University, Astana, Kazakhstan

Abstract:

We investigate the resonance behavior of soft biological cells by modeling their mechanical response as a damped nonlinear oscillator subject to external forcing. The governing equation is written as

$$m\,\ddot{u} + c\,\dot{u} + f_c = F(t),$$

where u represents the atomic force microscopy (AFM) indentation, f_c is the nonlinear contact restoring force, and F(t) is the applied excitation. We apply the Variational Iteration Method (VIM) and Pad'e approximation to derive closed-form expressions for the resonance frequencies. The frequencies are applied through the excitation to explore the nonlinear resonance behavior of the system. The results show that the oscillation amplitude and resonance characteristics depend strongly on the effective stiffness of the soft cell. As a representative example, the human breast cancer cell line MCF-7 is considered, with model parameters chosen to reflect effective cell mass, damping, and contact stiffness. Numerical analysis is used to examine forced vibrations generated by combinations of the oscillator's fundamental frequencies. The analysis reveals the emergence of beat-like amplitude modulation under near-resonant excitation, arising from the interaction between the driving frequency and the cell's intrinsic oscillatory modes. Damping effects are investigated in both transient free-decay motion and steady-state forced response. The proposed approach provides an efficient framework for understanding nonlinear resonance phenomena in soft-cell mechanics and may offer useful insight into the frequency-dependent mechanical behavior of cancer cells. The above results from the 1D model are compared favorably with COMSOL Multiphysics simulations of 3D cancer cell models.

Keywords:

Nonlinear oscillator, Resonance frequency, Soft cell mechanics, Atomic force microscopy, Variational iteration method, Padé approximation, MCF-7 cells, forced vibration, COMSOL Multiphysics.