Rolling Bodies, Trajectoids, and Time-Optimal Motion: A Mathematical Survey

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Abstract

This survey reviews the mathematical theory of rolling rigid bodies, with emphasis on trajectoids and the open problem of time-optimal rolling. We begin with the classical mechanics of rolling on an inclined plane and along circular arcs. We then examine the development of a geometric framework for rolling without slipping or spinning using concepts from differential geometry and geometric control theory. The plate-ball (sphere-on plane) system is treated as the canonical reference model, and the difficulties involved in extending spherical rolling models to general rolling bodies are discussed. We then extend the analysis to irregular rolling surfaces and review the construction of trajectoids, convex solids engineered to trace prescribed periodic paths under gravity, together with the associated existence results. We investigate how surface geometry, contact structure, and rolling constraints influence admissible motion in order to formulate the time-optimal rolling problem using the Pontryagin Maximum Principle. Trajectoids are interpreted as a special class of constrained irregular rolling bodies. The principal open problems are identified, and the mathematical obstacles to a complete solution are discussed. We conclude by identifying what is required for a complete treatment of trajectoid time-optimality, why such a treatment remains beyond current methods, and possible directions for future work.

Keywords

Trajectoids, nonholonomic rolling, plate–ball problem, geometric control theory, sub Riemannian geometry, rolling manifolds, shape operator, brachistochrone.